Int J Theor Phys (2007) 46: 2378–2383
DOI 10.1007/s10773-007-9355-9

Remote Preparation of a Two-Particle Entangled State via Two Tripartite *W* Entangled States

Xiao-Qi Xiao · Jin-Ming Liu

Published online: 5 May 2007

Abstract We present a scheme for remotely preparing a general two-particle entangled state via two tripartite W entangled states of different amplitudes. In this scheme one sender and two remote receivers are involved. The sender can help either one of the receivers to remotely reconstruct the original state with the aid of the other receiver's two single-particle orthogonal measurements. It is shown that by means of the method of the positive operator-valued measurement, our remote state preparation scheme can be achieved probabilistically.

Keywords Remote state preparation · W state · Positive operator-valued measurement

Entanglement, a new class of correlations between quantum systems making quantum information different from classical information in nature, is a fundamental conception in quantum mechanics and quantum information theory. It has provided a new physical resource for quantum information processing such as quantum teleportation [1], quantum cryptography [2], remote state preparation [3–8], quantum computation [9] and so on. As to tripartite quantum pure states, there are only two nonequivalent classes, the Greenberger–Horne–Zeilinger (GHZ) class and the W class [10, 11], under stochastic local operation and classical communication. Although the properties of the tripartite entangled states have not been fully understood, the tripartite GHZ state and W state have been applied in many aspects of quantum information [12–15], including remote state preparation (RSP) which is regarded as an interesting direction of quantum information. RSP has prospered since its inception [3–5], and is still in rapid development. It is also called "teleportation for a known state", for its task

This project supported by the National Key Basic Research and Development Program of China under Grant No. 2006CB921604 and the National Natural Science Foundation of China under Grant Nos. 60578050 and 10434060.

X.-Q. Xiao · J.-M. Liu (✉)
Key Laboratory of Optical and Magnetic Resonance Spectroscopy, Department of Physics, East China Normal University, Shanghai 200062, China
e-mail: jmliu@phy.ecnu.edu.cn

X.-Q. Xiao
Department of Physics, Shangrao Normal College, Shangrao 334000, China

is the same as that of quantum teleportation. Compared with teleportation [1], RSP exhibits the stronger trade-off between the required entanglement and the classical communication cost [5, 6] due to the sender's prior knowledge of the original state in the process of quantum communication.

Owing to its distinct feature, RSP has attracted extensive attention and various schemes for RSP have been reported in succession [4, 16–20]. As we know, most of the previous schemes for RSP discuss the quantum state which is transmitted from one sender to one receiver [4, 18, 19], and few of them apply the method of the positive operator-valued measurement (POVM) to remotely prepare a two-particle entangled state. In recent years the W states have been investigated at all points, which gradually make the features of such states clear. Note that the W states are not only useful for quantum information processing, but also important for the test of quantum nonlocality without inequality [21]. Moreover, many schemes have been presented for the generation of the W states in the systems of cavity quantum electrodynamics [22–24]. Using two tripartite W entangled states of different amplitudes as the quantum channel, in this paper we attempt to propose a scheme to remotely prepare a two-particle entangled state from the sender to either one of two receivers based on the POVM operator on an auxiliary particle. During the whole process, the classical communication cost in our scheme is less than that required in the corresponding scheme for teleportation [25] and the total success probability unequal for each receiver could approach to 1.0 under given conditions.

To begin with, the sender Alice has precise knowledge of a two-particle entangled state which is given by

$$|\Phi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{1}$$

where the parameter α is a real number, the parameter β is a complex number and they satisfy $|\alpha|^2 + |\beta|^2 = 1$. Suppose that Alice would like to remotely prepare the state $|\Phi\rangle$ in the location of either one of the two receivers Bob and Carol who do not have any knowledge of it at all, and that Alice, Bob and Carol initially share two tripartite entangled W states of different amplitudes, which can be expressed as

$$\begin{aligned} |\Psi\rangle_{1,2,3} &= a|100\rangle_{1,2,3} + b|010\rangle_{1,2,3} + c|001\rangle_{1,2,3}, \\ |\Psi\rangle_{4,5,6} &= r|100\rangle_{4,5,6} + s|010\rangle_{4,5,6} + t|001\rangle_{4,5,6}, \end{aligned} \tag{2}$$

where $|a|^2 + |b|^2 + |c|^2 = 1$, $|r|^2 + |s|^2 + |t|^2 = 1$, and particles 1, 4 belong to Alice, particles 2, 5 belong to Bob, and particles 3, 6 belong to Carol. Without loss of generality, we assume that the real parameters satisfy $a \geqslant b \geqslant c \geqslant 0$, $r \geqslant s \geqslant t \geqslant 0$. Therefore, the state of the whole system composed of the six particles can be written as

$$|\Psi\rangle_{\text{whole}} = |\Psi\rangle_{1,2,3} \otimes |\Psi\rangle_{4,5,6}. \tag{3}$$

To help either one of the receivers reestablish the original state $|\Phi\rangle$, Alice carries out a two-particle projective measurement on her particles 1 and 4 in a set of complete orthonormal bases of four-dimensional Hilbert space,

$$\begin{aligned} |\psi\rangle_{1,4} &= \alpha|00\rangle_{1,4} + \beta|11\rangle_{1,4}, \\ |\psi_\perp\rangle_{1,4} &= \beta^*|00\rangle_{1,4} - \alpha|11\rangle_{1,4}, \\ |\phi\rangle_{1,4} &= \alpha|01\rangle_{1,4} + \beta|10\rangle_{1,4}, \\ |\phi_\perp\rangle_{1,4} &= \beta^*|01\rangle_{1,4} - \alpha|10\rangle_{1,4}. \end{aligned} \tag{4}$$

After Alice's two-particle joint measurement, there will be the following four possible results

$$\begin{aligned}
{}_{1,4}\langle\psi|\Psi\rangle_{\text{whole}} &= \alpha(bs|1010\rangle_{2,3,5,6} + bt|1001\rangle_{2,3,5,6} + cs|0110\rangle_{2,3,5,6} + ct|0101\rangle_{2,3,5,6}) \\
&\quad + \beta^* ar|0000\rangle_{2,3,5,6}, \\
{}_{1,4}\langle\psi_\perp|\Psi\rangle_{\text{whole}} &= \beta(bs|1010\rangle_{2,3,5,6} + bt|1001\rangle_{2,3,5,6} + cs|0110\rangle_{2,3,5,6} + ct|0101\rangle_{2,3,5,6}) \\
&\quad - \alpha ar|0000\rangle_{2,3,5,6}, \\
{}_{1,4}\langle\phi|\Psi\rangle_{\text{whole}} &= \alpha(br|1000\rangle_{2,3,5,6} + cr|0100\rangle_{2,3,5,6}) + \beta^*(as|0010\rangle_{2,3,5,6} + at|0001\rangle_{2,3,5,6}), \\
{}_{1,4}\langle\phi_\perp|\Psi\rangle_{\text{whole}} &= \beta(br|1000\rangle_{2,3,5,6} + cr|0100\rangle_{2,3,5,6}) - \alpha(as|0010\rangle_{2,3,5,6} + at|0001\rangle_{2,3,5,6}),
\end{aligned} \tag{5}$$

where the normalization factors are discarded.

If the result Alice gets is $|\psi_\perp\rangle_{1,4}$, then the remaining particles will collapse into the state

$$\begin{aligned}
|\Psi\rangle_{2,3,5,6} &= \beta(bs|1010\rangle_{2,3,5,6} + bt|1001\rangle_{2,3,5,6} + cs|0110\rangle_{2,3,5,6} + ct|0101\rangle_{2,3,5,6}) \\
&\quad - \alpha ar|0000\rangle_{2,3,5,6}.
\end{aligned} \tag{6}$$

Provided that Alice intends to reconstruct the original state in Bob's location, she sends the measurement result to Bob through a classical channel. In this instance, Carol is asked to perform two single-particle projective measurements on particles 3 and 6 on the basis of $\{|0\rangle, |1\rangle\}$, respectively, we then have

$$\begin{aligned}
|\Psi\rangle_{2,3,5,6} &= (\beta bs|11\rangle_{2,5} - \alpha ar|00\rangle_{2,5})|00\rangle_{3,6} \\
&\quad + \beta bt|1001\rangle_{2,3,5,6} + \beta cs|0110\rangle_{2,3,5,6} + \beta ct|0101\rangle_{2,3,5,6},
\end{aligned} \tag{7}$$

which is unnormalized. Obviously, it is possible for the RSP process to be successful only if Carol gets the result $|00\rangle_{3,6}$, otherwise the RSP scheme will fail. After Carol's measurement result is transmitted to Bob through the classical channel, he introduces an auxiliary two-state particle F which is initially in the state $|0\rangle_F$, and implements a controlled-not operation (CNOT) with particle 2 (or particle 5) as the control bit and particle F as the target bit. Then the state combined by particles 2, 5 and F will become

$$\begin{aligned}
&\frac{1}{\sqrt{|\beta|^2 b^2 s^2 + \alpha^2 a^2 r^2}}(\beta bs|11\rangle_{2,5} - \alpha ar|00\rangle_{2,5})|0\rangle_F \quad \xrightarrow{\text{CNOT}} \\
&|\Psi\rangle_{2,5,F} = \frac{1}{\sqrt{|\beta|^2 b^2 s^2 + \alpha^2 a^2 r^2}}(\beta bs|111\rangle_{2,5,F} - \alpha ar|000\rangle_{2,5,F}) \\
&\qquad = -\frac{\sqrt{a^2 r^2 + b^2 s^2}}{2\sqrt{|\beta|^2 b^2 s^2 + \alpha^2 a^2 r^2}}[(\alpha|00\rangle_{2,5} + \beta|11\rangle_{2,5})(x_1|0\rangle_F - y_1|1\rangle_F) \\
&\qquad\quad + (\alpha|00\rangle_{2,5} - \beta|11\rangle_{2,5})(x_1|0\rangle_F + y_1|1\rangle_F)],
\end{aligned} \tag{8}$$

where $x_1 = \frac{ar}{\sqrt{a^2r^2+b^2s^2}}$ and $y_1 = \frac{bs}{\sqrt{a^2r^2+b^2s^2}}$ $(x_1 \geqslant y_1)$. To achieve the RSP scheme, we need to discriminate between the two nonorthogonal quantum states $x_1|0\rangle_F - y_1|1\rangle_F$ and $x_1|0\rangle_F + y_1|1\rangle_F$. According to Ref. [26], such two nonorthogonal states can be identified by an optimal positive operator valued measurement (POVM) on the auxiliary particle F, which is described as

$$A_1=\frac{1}{2x_1^2}\begin{pmatrix} y_1^2 & x_1y_1 \\ x_1y_1 & x_1^2 \end{pmatrix},\qquad A_2=\frac{1}{2x_1^2}\begin{pmatrix} y_1^2 & -x_1y_1 \\ -x_1y_1 & x_1^2 \end{pmatrix},\qquad A_3=\begin{pmatrix} 1-\frac{y_1^2}{x_1^2} & 0 \\ 0 & 0 \end{pmatrix}. \tag{9}$$

If the result of his measurement is A_1, Bob can conclude that the state of particles 2 and 5 must be $\alpha|00\rangle_{2,5}-\beta|11\rangle_{2,5}$ which can be transformed into the original state $|\Phi\rangle$ with a unitary transformation $\hat{\sigma}_z$ on particle 2 (or particle 5). If the measurement outcome A_2 occurs, then the state Bob obtains will be the state to be transmitted remotely. It is easy to verify that no matter whether the result is A_1 or A_2, the success probability to obtain the desired result is $(2y_1^2)$. In this case, the probability of successful remote state preparation for the receiver Bob is

$$|_{3,6}\langle 00|_{1,4}\langle\psi_\perp|\Psi\rangle_{\text{whole}}|^2\cdot(1-{}_{2,5,F}\langle\Psi|I_2\otimes I_5\otimes A_3|\Psi\rangle_{2,5,F})=(bs)^2, \tag{10}$$

where I denotes an identity operator. Similarly, the state shown in (1) can also be reestablished at Carol's side. At this stage, Alice's measurement result should be transferred to Carol, and Bob needs to send his measurement result to Carol after he performs two single-particle projective measurements on his two particles 2 and 5, respectively. By performing similar operations as Bob does above, Carol can reconstruct the initial state $|\Phi\rangle$ with the success probability $(ct)^2$. By analogical deduction, when the outcome of Alice's measurement is $|\phi_\perp\rangle_{1,4}$, the RSP scheme can be realized in either location of the two receivers and the corresponding success probability would be $(bs)^2$ at Bob's side or $(ct)^2$ at Carol's side.

Gathering Alice's two measurement results $|\psi_\perp\rangle_{1,4}$ and $|\phi_\perp\rangle_{1,4}$ together, we find that the two-particle entangled state $|\Phi\rangle$ can be restored at either one of the receivers' sides with a different success probability. To be exact, the success probability is $2(bs)^2$ if the original state is reproduced in Bob's location while the success probability is $2(ct)^2$ if the initial state is reestablished at Carol's side. Only if $a=b=c=\frac{1}{\sqrt{3}}$, $r=s=t=\frac{1}{\sqrt{3}}$, *i.e.*, the quantum channel is composed of two W states of same amplitudes

$$|\Psi\rangle=\frac{1}{\sqrt{3}}(|100\rangle+|010\rangle+|001\rangle), \tag{11}$$

each receiver has the same success probability which is $\frac{2}{9}$.

Besides, we should emphasize that according to the (5), if the measurement result Alice obtains is $|\psi\rangle_{1,4}$ or $|\phi\rangle_{1,4}$, the RSP scheme will fail because neither of the two receivers has knowledge of the parameters of the state $|\Phi\rangle$, and they cannot convert the states of their particles into the original state $|\Phi\rangle$ due to the involvement of an antiunitary operation [4]. Yet, there could be some exceptions. When the original state is one of some specially chosen bipartite entangled states in which the parameters α, β are all real or when it is in the form of $|\Phi\rangle=\frac{1}{\sqrt{2}}(|00\rangle+e^{i\varphi}|11\rangle)$, remote preparation of these two-particle entangled states can be probabilistically realized at the side of either one of the two receivers even if the outcome of Alice's measurement is $|\phi\rangle_{1,4}$ or $|\psi\rangle_{1,4}$. For example, we consider the case that the parameters α, β of the original state $|\Phi\rangle$ are all real and the result of the joint measurement performing on Alice's particles is $|\phi\rangle_{1,4}$, at this moment the remaining particles will be projected to the state

$$|\Psi\rangle_{2,3,5,6}=\alpha(br|1000\rangle_{2,3,5,6}+cr|0100\rangle_{2,3,5,6})+\beta(as|0010\rangle_{2,3,5,6}+at|0001\rangle_{2,3,5,6}). \tag{12}$$

Next we assume that the original state will be reconstructed at Bob's side. After Carol can assure him of the fact that particles 3 and 6 are in the state $|00\rangle_{3,6}$ through two separately single-particle projective measurements, Bob introduces an auxiliary qubit $|0\rangle_F$. Now he carries out a CNOT unitary operation between particles 2 and F with the particle 2 being the controller and the particle F as the target, then we have

$$\begin{aligned}
&\frac{1}{\sqrt{\alpha^2b^2r^2+\beta^2a^2s^2}}(\alpha br|10\rangle_{2,5}+\beta as|01\rangle_{2,5})|0\rangle_F \quad \stackrel{\text{CNOT}}{\longrightarrow}\\
&\frac{1}{\sqrt{\alpha^2b^2r^2+\beta^2a^2s^2}}(\alpha br|101\rangle_{2,5,F}+\beta as|010\rangle_{2,5,F})\\
&=\frac{\sqrt{b^2r^2+a^2s^2}}{2\sqrt{\alpha^2b^2r^2+\beta^2a^2s^2}}[(\alpha|10\rangle_{2,5}+\beta|01\rangle_{2,5})(x_2|1\rangle_F+y_2|0\rangle_F)\\
&\quad+(\alpha|10\rangle_{2,5}-\beta|01\rangle_{2,5})(x_2|1\rangle_F-y_2|0\rangle_F)],
\end{aligned} \tag{13}$$

where $x_2=\frac{br}{\sqrt{a^2s^2+b^2r^2}}$ and $y_2=\frac{as}{\sqrt{a^2s^2+b^2r^2}}$. To realize the remote preparation of the state $|\Phi\rangle$ at Bob's side, it is necessary to distinguish the state $x_2|1\rangle_F+y_2|0\rangle_F$ from the state $x_2|1\rangle_F-y_2|0\rangle_F$ by virtue of the following POVM operation on particle F,

$$\begin{aligned}
&B_1=\frac{\mu^2}{2}\begin{pmatrix}\frac{1}{x_2^2} & \frac{1}{x_2y_2}\\ \frac{1}{x_2y_2} & \frac{1}{y_2^2}\end{pmatrix}, \qquad B_2=\frac{\mu^2}{2}\begin{pmatrix}\frac{1}{x_2^2} & -\frac{1}{x_2y_2}\\ -\frac{1}{x_2y_2} & \frac{1}{y_2^2}\end{pmatrix},\\
&B_3=\begin{pmatrix}1-\frac{\mu^2}{x_2^2} & 0\\ 0 & 1-\frac{\mu^2}{y_2^2}\end{pmatrix},
\end{aligned} \tag{14}$$

where the coefficient $\mu^2=\frac{b^2s^2}{a^2s^2+b^2r^2}$. Generally, this kind of generalized measurement given in the above equation is different from the optimal POVM described by (9) since its conclusive probability of the measurement on the auxiliary particle does not attain maximum. But based on the result of the POVM on the particle F, Bob can also infer the state of the particles 2 and 5. As a result, he can reestablish the original state $|\Phi\rangle$ with success probability $(bs)^2$ by preforming a σ_x operator on particle 2 and an identity transformation I on particle 5 if the particles 2 and 5 are in the state $\alpha|10\rangle_{2,5}+\beta|01\rangle_{2,5}$, or by preforming a $i\hat{\sigma}_y$ unitary operation on particle 2 if the particles 2 and 5 are in the state $\alpha|10\rangle_{2,5}-\beta|01\rangle_{2,5}$. With the similar method, when the outcome of Alice's projective measurement is $|\psi\rangle_{1,4}$, remote preparation of some special ensembles of two-particle entangled states can be realized at Bob's side with success probability $(bs)^2$. Therefore, together with the four results of Alice's measurement on particles 1 and 4 obtained above, some specially bipartite entangled states can be reproduced in Bob's location with the total success probability $4(bs)^2$, which is equal to $\frac{4}{9}$ if the quantum channel consists of two W states given by (11). We notice that the total success probability will approach to 1.0 when the parameters satisfy $a\approx b\approx\frac{1}{\sqrt{2}}$, $c\approx 0$ and $r\approx s\approx\frac{1}{\sqrt{2}}$, $t\approx 0$. In fact, this case is equivalent to the case of the RSP scheme via two EPR states discussed in [20]. By similar analyses, we deduce that these specially chosen initial states to be prepared remotely can also be reestablished at Carol's side with the total success probability $4(ct)^2$.

In summary, we have presented a scheme for the remote preparation of a general two-particle entangled state from the sender Alice to either one of the two receivers Bob and

Carol via two tripartite W entangled states of different amplitudes. It is shown that no matter whether the original state is needed to reestablish at Bob's side or at Carol's side, a POVM operation is introduced to distinguish between two nonorthogonal quantum states. We have found that the total success probability of RSP, the maximum of which can approach to 1, is dependent on the parameters of the channel described by (1), and is different for each receiver unless the quantum channel consists of two W states. Furthermore, four bits of classical communication, which are less than those needed in the corresponding teleportation scheme [25], are enough to achieve our RSP scheme. Unlike the previous schemes discussing the RSP only from one sender to one receiver [4, 18, 19], in principle the original state to be prepared remotely in our scheme can be reconstructed at either side of the two receivers who are distantly separated from each other. Besides, the present scheme can be directly generalized to remotely prepare a general two-particle entangled state from a sender to any one of N receivers by using two $(N+1)$-particle partially entangled W states. Since the three-photon polarization entangled W states [27] and the POVM operation [28–30] have been observed experimentally in optical systems, our proposed scheme might be realizable with current techniques.

References

1. Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
2. Ekert, A.: Phys. Rev. Lett. **67**, 661 (1991)
3. Lo, H.K.: Phys. Rev. A **62**, 012313 (2000)
4. Pati, A.K.: Phys. Rev. A **63**, 014302 (2001)
5. Bennett, C.H., DiVincenzo, D.P., Shor, P.W., Smolin, J.A., Terhal, B.M., Wootters, W.K.: Phys. Rev. Lett. **87**, 077902 (2001)
6. Devetak, I., Berger, T.: Phys. Rev. Lett. **87**, 197901 (2001)
7. Berry, D.W.: Phys. Rev. A **70**, 062306 (2004)
8. Ye, M.-Y., Zhang, Y.-S., Guo, G.-C.: Phys. Rev. A **69**, 022310 (2004)
9. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information, pp. 171–211. Cambridge University Press, New York (2000)
10. Dur, W., Vidal, G., Cirac, J.I.: Phys. Rev. A **62**, 062314 (2000)
11. Acin, A., Bruß, D., Lewenstein, M., Sanpera, A.: Phys. Rev. Lett. **87**, 040401 (2001)
12. Yu, Y.F., Feng, J., Zhan, M.S.: Phys. Rev. A **68**, 024303 (2003)
13. Wu, X.H., Zong, H.S., Pang, H.R.: Phys. Lett. A **276**, 221 (2000)
14. Ye, L., Yu, L.B.: Phys. Lett. A **346**, 330 (2005)
15. Zheng, S.B.: Phys. Rev. A **70**, 045804 (2004)
16. Yu, C.S., Song, H.S., Wang, Y.H.: Phys. Rev. A **73**, 022340 (2006)
17. Babichev, S.A., Brezger, B., Lvovsky, A.I.: Phys. Rev. Lett. **92**, 047903 (2004)
18. Zeng, B., Zhang, P.: Phys. Rev. A **65**, 022316 (2002)
19. Shi, B.S., Tomita, A.: J. Opt. B: Quantum Semiclass. Opt **4**, 380 (2002)
20. Liu, J.M., Wang, Y.Z.: Phys. Lett. A **316**, 159 (2003)
21. Zheng, S.B.: Phys. Rev. A **66**, 014103 (2002)
22. Guo, G.C., Zhang, Y.S.: Phys. Rev. A **65**, 054302 (2002)
23. Zheng, S.B.: J. Opt. B: Quantum Semiclass. Opt **7**, 10 (2005)
24. Zheng, S.B.: J. Phys. B: At. Mol. Opt. Phys. **39**, 2505 (2006)
25. Dai, H.Y., Chen, P.X., Li, C.Z.: J. Opt. B: Quantum Semiclass. Opt **6**, 106 (2004)
26. Mor, T., Horodecki, P.: quant-ph/9906039 (1999)
27. Manfred, E., Kiesel, N., Bourennane, M., Kurtsiefer, C., Weinfurter, H.: Phys. Rev. Lett. **92**, 077901 (2004)
28. Huttner, B., Muller, A., Gautier, J.D., Zbinden, H., Gisin, N.: Phys. Rev. A **54**, 3789 (1996)
29. Clarke, R.B.M., Chefles, A., Barnett, S.M., Riis, E.: Phys. Rev. A **63**, 040305 (2001)
30. Mohseni, M., Steinberg, A.M., Bergou, J.A.: Phys. Rev. Lett. **93**, 200403 (2004)